# NATIONAL SENIOR CERTIFICATE 

## GRADE 12

## SEPTEMBER 2014

## MATHEMATICS P2

MARKS: 150

TIME: 3 hours


This question paper consists of 15 pages, including 2 diagram sheets and 1 information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of ELEVEN questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and nongraphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. TWO diagram sheets for QUESTION 2.1, QUESTION 2.3, QUESTION 8.2 and QUESTION 9.1 are attached at the end of this question paper. Write your name on these sheets in the spaces provided and attach it to your answer book.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

## QUESTION 1

1.1 Ros is the sales manager for a print and design company. His staff members must report about the number of clients they visit every month. Below is the ogive representing the collected data in one month.

1.1 How many staff members were there?
1.2 Determine the semi-interquartile range.
1.3 Draw a box whisker diagram for the data.
1.4 Comment on the distribution of data using your result in QUESTION 1.3.

## QUESTION 2

The data below shows the marks obtained by ten Grade 12 learners from two different Mathematics classes sitting in the first row of each class.

| Class A | 16 | 36 | 20 | 38 | 40 | 30 | 35 | 22 | 40 | 24 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Class B | 45 | 70 | 44 | 56 | 60 | 48 | 75 | 60 | 63 | 38 |

2.1 Make use of the grid provided on the DIAGRAM SHEET 1 to draw a scatter plot for the data.
2.2 Calculate the equation of the least squares regression line for this data.
2.3 Draw the least squares regression line for the data on the scatter plot diagram drawn in QUESTION 2.1 (DIAGRAM SHEET 1).
2.4 Calculate the correlation coefficient for the above data.
2.5 Calculate the mean and the standard deviation for Class B.
2.6 How many scores fall within one standard deviation of the mean?

## QUESTION 3

In the figure, $\mathrm{A}(0 ; 9), \mathrm{B}(11 ; 9)$ and $\mathrm{C}(13 ; 1)$ are the vertices of $\Delta \mathrm{ABC}$ with altitude CD .

3.1 Write down the equation of the line AB .
3.2 Write down the length of AB.
3.3 Determine the co-ordinates of D.
3.4 Calculate the area of $\triangle \mathrm{ABC}$.
3.5 Determine the co-ordinates of M , the midpoint of AC .
3.6 Hence, determine the equation of the perpendicular bisector of AC.
3.7 Does the perpendicular bisector in QUESTION 3.6 pass through point B or not? Justify your answer using relevant calculations.
3.8 Determine $\angle \mathrm{ABC}$ correct to the nearest degree.
3.9 Determine the equation of a line parallel to AC and passing through D .

## QUESTION 4

The length of the radius of the circle with equation: $x^{2}-2 x+y^{2}+4 y=a$ is 5 units.

4.1 Show by means of calculations that $a=20$ units.
4.2 Write down the coordinates of the centre M of the circle.
4.3 $\mathrm{A}(x ; y)$, with $y>0$, is one of the points of intersection of the circle and the straight line $x=4$. Determine the values of $x$ and $y$.
4.4 Determine the equation of the tangent to the circle at the point A .
4.5 Determine whether the point $\mathrm{T}(-1 ;-2)$ lies inside or outside the circle.
4.6 If the circle is translated 3 units to the left and 1 unit up, determine the equation of the new circle.

## QUESTION 5

5.1 If $\cos 62^{\circ}=m$, determine the value of each of the following in terms of $m$.
5.1.1 $\sin 28^{\circ}$
5.1.2 $\cos 362^{\circ}$
5.2 Simplify to a single ratio:

$$
\begin{equation*}
\frac{\tan \left(360^{\circ}-x\right) \cdot \sin \left(90^{\circ}+x\right)}{\sin (-x)} \tag{5}
\end{equation*}
$$

5.3 If $4 \sin ^{2} \theta-3=0,90^{\circ} \leq \theta \leq 180^{\circ}$, calculate the value of:

$$
\begin{equation*}
\cos \frac{1}{4} \theta \cdot \sin \frac{1}{2} \theta-\tan \left(3 \theta-45^{\circ}\right) \tag{6}
\end{equation*}
$$

## QUESTION 6

6.1 Prove that $\frac{\cos 2 x}{1+\sin 2 x}=\frac{\cos x-\sin x}{\sin x+\cos x}$

## QUESTION 7

7.1 Use the diagram on DIAGRAM SHEET to prove that:

$$
a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A
$$


7.2 In the figure, the diameter PQ of the circle is produced to $\mathrm{R} . \mathrm{S}$ is a point on the circumference such that $\mathrm{QR}=\mathrm{QS}=x$. $P$ and $S$ are joined and $\mathrm{Q} \hat{P} S=\alpha$.

7.2.1 Prove that : $\mathrm{SR}=x \sqrt{2(1+\sin \propto)}$
7.2.2 If $\mathrm{SR}=5 \sqrt{3}$ and $x=5$, show that $\mathrm{PQ}=10$

## QUESTION 8

Given $f(x)=2 \cos x$ and $g(x)=\sin \left(x+30^{\circ}\right)$
8.1 Write down the amplitude of $f$.
8.2 Sketch the graph of $f$ and $g$ on the same set of axes on DIAGRAM SHEET 2.
8.3 Determine the value(s) of $x$ for which $f(x)-g(x) \geq 0$ for $x \in\left[-180^{\circ} ; 90^{\circ}\right]$.
8.4 Give TWO possible ways of transforming the graph of $f$ such that its $y$-intercept is at the origin.

## QUESTION 9

9.1 O is the centre of the circle and $\mathrm{A}, \mathrm{D}$ and C are on the circumference of the circle. Use the diagram to prove, USING Euclidean geometry methods, the theorem which states that $\angle A O D=2 \angle A C D$.

9.2 In the figure, M is the centre of the circle through $\mathrm{A}, \mathrm{B}$, and C . OB and AC bisect $M \hat{B} C$ and $\mathrm{M} \hat{C} \mathrm{~B}$ respectively with O on $\mathrm{AC} . \mathrm{AB}$ is joined.


Let $\hat{B}_{2}=x$
9.2.1 Determine the size of $\widehat{O}_{2}$ in terms of x .
9.2.2 Prove that AO is a diameter of circle ABO .
9.3 In the diagram below, EF is the chord of a circle with centre O. OG is perpendicular to EF.


If $\mathrm{OE}=x \mathrm{~cm}$ and $\mathrm{HG}=2 \mathrm{~cm}$, determine with reasons the size of EF (in its simplest form) in terms of $x$.

## QUESTION 10

In the diagram below, O is the centre with $\mathrm{A}, \mathrm{B}$ and T on the circumference, $\mathrm{BP}=\mathrm{OB}=\mathrm{AO}, \mathrm{PTR}$ is a tangent and $\mathrm{EP} \perp \mathrm{AP}$.


Prove that:

### 10.1 TEPB is a cyclic quad

$10.2 \Delta \mathrm{ATB} / / / \Delta \mathrm{APE}$
10.3 TP = PE
$10.4 \Delta \mathrm{ATB} / / / \Delta \mathrm{EPB}$
$10.5 \quad 2 \mathrm{BP}^{2}=$ BE. TB

## QUESTION 11

11.1 Complete the following statement:

A line parallel to one side of a triangle divides ...
11.2 In $\Delta \mathrm{KLM}, \widehat{K}=90^{\circ}$ and D is a point on KL so that $\mathrm{KD}: \mathrm{DL}=2: 1$ and $\mathrm{DE} / / \mathrm{LM}$ with E on KM .

11.2.1 If $\mathrm{DL}=x$ and $\mathrm{EM}=y$, express $\mathrm{LM}^{2}$ in terms of $x$ and $y$.
11.2.2 Show that $\mathrm{DM}^{2}+\mathrm{LE}^{2}=\frac{13}{9} \mathrm{LM}^{2}$.

TOTAL: 150

SURNAME: ........................................ NAME:

## DIAGRAM SHEET 1

QUESTIONS 2.1 AND 2.3


## QUESTION 7.1



SURNAME:
NAME: $\qquad$

## DIAGRAM SHEET 2

QUESTION 8.2


## QUESTION 9.1



## INFORMATION SHEET: MATHEMATICS

$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
$A=P(1+n i) \quad A=P(1-n i) \quad A=P(1-i)^{n} \quad A=P(1+i)^{n}$
$T_{n}=a+(n-1) d \quad \mathrm{~S}_{n}=\frac{n}{2}(2 a+(n-1) d)$
$T_{n}=a r^{n-1} \quad S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} ; r \neq 1 \quad S_{\infty}=\frac{a}{1-r} ;-1<r<1$
$F=\frac{x\left[(1+i)^{n}-1\right]}{i} \quad P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$
$f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \quad \mathrm{M}\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
$y=m x+c \quad y-y_{1}=m\left(x-x_{1}\right) \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m=\tan \theta$
$(x-a)^{2}+(y-b)^{2}=r^{2}$
In $\triangle A B C: \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A \quad$ area $\triangle A B C=\frac{1}{2} a b \cdot \sin C$
$\sin (\alpha+\beta)=\sin \alpha \cdot \cos \beta+\cos \alpha \cdot \sin \beta \quad \sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta$
$\cos (\alpha+\beta)=\cos \alpha \cdot \cos \beta-\sin \alpha \cdot \sin \beta \quad \cos (\alpha-\beta)=\cos \alpha \cdot \cos \beta+\sin \alpha \cdot \sin \beta$
$\cos 2 \alpha=\left\{\begin{array}{l}\cos ^{2} \alpha-\sin ^{2} \alpha \\ 1-2 \sin ^{2} \alpha \\ 2 \cos ^{2} \alpha-1\end{array} \quad \sin 2 \alpha=2 \sin \alpha \cdot \cos \alpha\right.$
$\bar{x}=\frac{\sum f x}{n}$

$$
\sigma^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}
$$

$P(A)=\frac{n(A)}{n(S)}$
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
$\hat{y}=a+b x$

$$
b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}}
$$

